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A Deviation Function Used for the Secondary Realization of the ITS-90 in the Subrange from 83.8033 K to 273.16 K

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Abstract This article proposes a correlation relation between the resistance ratios of the triple points of argon and mercury. By this relation, the resistance ratio of the triple point of argon can be extrapolated from that of mercury, and a deviation function which is defined in the range from 83.8058 K to 273.16 K can be determined from only the calibration values at the triple points of water and mercury. It is a close approximation to the ITS-90 deviation function in the subrange. Using it, the calibration at the triple point of argon can be saved. Twenty-five standard platinum resistance thermometers are used to check the function. The errors are less than 5 mK. It is sufficient for secondary measurements.

Keywords Deviation functions · Extrapolated function · Secondary measurement · Standard platinum resistance thermometers (SPRTs)

1 Introduction

Standard platinum resistance thermometers (SPRTs) as ITS-90 interpolation instruments have some properties different from other thermometers. One of the properties is that the difference between resistances of the SPRTs is very small, and it leads to

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a correlation between the calibration values at some fixed points to some extent [1]. This article proposes a correlative relation between the resistance ratios of the triple points of argon and mercury in a way similar to that in [2]. By this relation, the resistance ratio of the triple point of argon can be extrapolated from that of mercury, and a deviation function which is defined in the range from 83.8058 K to 273.16 K can be determined from only the calibration values at the triple points of water and mercury.

2 Correlative Relation and Its Application

The correlative relation given by this article is

$$W_{\mathrm{Ar}}^{*} = 5.031123288 W_{\mathrm{Hg}} - 4.031123288 \tag{1}$$

where W_{Hg} expresses the resistance ratio at the triple point of mercury, W_{Ar}^{*} is an approximation value of the resistance ratio at the triple point of argon, and W_{Ar} and $*$ are used to distinguish W_{Ar}^{*} from W_{Ar}.

In actual application, substituting W_{Hg} into Eq. 1, W_{Ar}^{*} can be obtained. Then, they are substitute in the deviation function,

$$W(T) - W_{\mathrm{r}}(T) = a[W(T) - 1] + b(W(T) - 1] \ln W(T) \tag{2}$$

in the subrange from 83.8058 K to 273.16 K given by ITS-90, and a linear set of equations is established. Solving the set of equations, the coefficients a, b and the deviation function can be obtained.

3 Basis of Approximation Relation

The correlative relation is based on

$$\frac{W_{\mathrm{Ar}} - W_{\mathrm{Hg}}}{W_{\mathrm{Hg}} - 1} = \frac{W_{\mathrm{r}}(\mathrm{Ar}) - W_{\mathrm{r}}(\mathrm{Hg})}{W_{\mathrm{r}}(\mathrm{Hg}) - 1}(1 + \alpha) \tag{3}$$

where $W_{\mathrm{r}}(\mathrm{Ar})$ and $W_{\mathrm{r}}(\mathrm{Hg})$ represent the reference function values at the fixed points of argon and mercury, respectively, the number 1 can be considered as the resistance ratio at the triple point of water, and α is a linear coefficient much smaller than 1. In Eq. 3, letting $\alpha = 0$ and W_{Ar} replaced with its approximation W_{Ar}^{*}, it becomes

$$\frac{W_{\mathrm{Ar}}^{*} - W_{\mathrm{Hg}}}{W_{\mathrm{Hg}} - 1} = \frac{W_{\mathrm{r}}(\mathrm{Ar}) - W_{\mathrm{r}}(\mathrm{Hg})}{W_{\mathrm{r}}(\mathrm{Hg}) - 1} \tag{4}$$

Substituting $W_{\mathrm{r}}(\mathrm{Ar}) = 0.21585974$ and $W_{\mathrm{r}}(\mathrm{Hg}) = 0.84414211$ into Eq. 4 yields Eq. 1.

Equation 3 reflects high consistency of the resistance temperature curves of SPRTs. On this manner, in order to reduce the number of fixed points and the order of the interpolation equation, the ITS-90 provides the resistance ratio of a real SPRT as the reference function and build a set of deviation functions,

$$\Delta W(T) = W(T) - W_r(T) \tag{5}$$

in the SPRT ranges. At the triple points of argon and mercury, specifically the deviations can be expressed as

$$\begin{aligned} W_{Ar} &= W_r(Ar) + \Delta W_{Ar} \\ W_{Hg} &= W_r(Hg) + \Delta W_{Hg} \end{aligned} \tag{6}$$

Thus, we have

$$\begin{aligned} \frac{W_{Ar} - W_{Hg}}{W_{Hg} - 1} &= \frac{W_r(Ar) - W_r(Hg) + \Delta W_{Ar} - \Delta W_{Hg}}{W_r(Hg) - 1 + \Delta W_{Hg}} \\ &= \frac{W_r(Ar) - W_r(Hg)}{W_r(Hg) - 1} \left(\frac{1 + \frac{\Delta W_{Ar} - \Delta W_{Hg}}{W_r(Ar) - W_r(Hg)}}{1 + \frac{\Delta W_{Hg}}{W_r(Hg) - 1}} \right) \\ &= \frac{W_r(Ar) - W_r(Hg)}{W_r(Hg) - 1} \left(\frac{1 + \beta}{1 + \gamma} \right) \\ &= \frac{W_r(Ar) - W_r(Hg)}{W_r(Hg) - 1} \left(1 + \frac{\beta - \gamma}{1 + \gamma} \right) \\ &= \frac{W_r(Ar) - W_r(Hg)}{W_r(Hg) - 1} (1 + \alpha) \end{aligned} \tag{7}$$

where

$$\begin{aligned} \beta &= \frac{\Delta W_{Ar} - \Delta W_{Hg}}{W_r(Ar) - W_r(Hg)} \\ \gamma &= \frac{\Delta W_{Hg}}{W_r(Hg) - 1} \\ \alpha &= \frac{\beta - \gamma}{1 + \gamma} \end{aligned} \tag{8}$$

A number of experiments show that the deviations, ΔW_{Ar} and ΔW_{Hg}, are smaller than 10^{-3} and much smaller than the reference function values $W_r(Ar)$, $W_r(Hg)$, and their difference, and β, γ, and α are much smaller than 1.

4 Checking

Twenty-five SPRTs, 12 in the range from 83.8058 K to 933.473 K and 13 in the subrange from 13.8133 K to 273.16 K, are used to check Eq. 1. The maximum error of SPRT No. 1872174 is 4.9 mK. The calibration data, approximation values, and errors are given in Tables 1 and 2, respectively.

Table 1 Calibration data of SPRTs

No.	SPRT no.	Normal resistance values (Ω)	W_{Hg}	W_{Ar}
1	1886904	25	0.844165636	0.215974765
2	1041	25	0.844162451	0.215944142
3	1857277	25	0.84416905	0.21598525
4	186095	25	0.84415967	0.21594546
5	1886906	25	0.8441615	0.2159495
6	1043	25	0.8441574	0.2159397
7	1774095	25	0.844180751	0.216041
8	1774092	25	0.844147257	0.2158770
9	1842381	25	0.8441474	0.2158865
10	1842397	25	0.8441476	0.2158860
11	1872174	25	0.844162509	0.215944117
12	213865	25	0.844152407	0.215911188
13	1728839	25	0.844146812	0.215886386
14	Hart 1094		0.84415147	0.21591058
15	Hart 1098A		0.84415159	0.21590922
16	Hart 1098B		0.84415119	0.21590928
17	Rosemount4386		0.84415929	0.21595356
18	Hart 1032		0.84415156	0.21591388
19	Rosemount4385		0.84415766	0.21594709
20	Hart 1030A		0.84415102	0.21590839
21	Hart 1030B		0.84415119	0.21590809
22	162 ce s/n 4727		0.84416360	0.2159706
23	5681with 1207		0.8441521	0.2159150
24	7906	25	0.84419846	0.21612827
25	729536	25	0.84416823	0.21599573

Note: The first 13 data come from Ref. [3], data Nos. 14–21 from Ref. [4], data Nos. 22–23 from PTB, and the last two data from Ref. [5]

5 Error Analysis

The error of the deviation function given in this article results from the approximation calibration values at the triple point of argon which only cause changes of the interpolation coefficients of the ITS-90 deviation function Eq. 2, and the changes propagate to any temperature in the subrange from 83.8058 K to 273.16 K. According to this, the error function has the form

$$\delta W(T) = W(T)^* - W(T) = \delta a[W(T) - 1] + \delta b[W(T) - 1] \ln W(T) \qquad (9)$$

where $\delta W(T)$ represents an error function, $W(T)^*$ represents the approximating deviation function values, and δa and δb are the changes of coefficients a and b of Eq. 2.

Table 2 Approximation values and errors at the triple point of argon

No.	SPRT no.	W^*_{Ar}	$\left(W^*_{\text{Ar}} - W_{\text{Ar}}\right) 10^6$	Error (mK)
1	1886904	0.21597819	3.425	0.79
2	1041	0.215962116	18.024	4.2
3	1857277	0.215995366	10.106	2.3
4	186095	0.215948174	2.714	0.63
5	1886906	0.215957381	7.881	1.8
6	1043	0.215936375	−2.946	−0.68
7	1774095	0.216054235	13.235	3.05
8	1774092	0.215885635	8.635	2.0
9	1842381	0.215886354	−0.145	−0.033
10	1842397	0.21588736	1.36	0.31
11	1872174	0.215962396	21.199	4.9
12	213865	0.215909743	−1.453	−0.34
13	1728839	0.215883396	−2.989	−0.69
14	Hart 1094	0.215906831	−3.748	−0.86
15	Hart 1098A	0.215907435	−1.748	−0.41
16	Hart 1098B	0.215905422	−3.857	−0.89
17	Rosemount4386	0.215946147	−7.385	−1.7
18	Hart 1032	0.215907284	−6.595	−1.5
19	Rosemount4385	0.215937974	−9.116	−2.1
20	Hart 1030A	0.215904567	−3.822	−0.88
21	Hart 1030B	0.215905422	−0.614	−0.61
22	162 ce s/n 4727	0.215967858	−2.741	−0.63
23	5681with 1207	0.2159100	−5.000	−1.2
24	7906	0.216143243	14.973	3.5
25	729536	0.215991152	−1.054	−1.1

Both $W(T)^*$ and $W(T)$ passing through the triple point of mercury result in

$$\delta W_{\text{Hg}} = \delta a[W_{\text{Hg}} - 1] + \delta b[W_{\text{Hg}} - 1] \ln W_{\text{Hg}} = 0$$

Noting that $W_{\text{Hg}} - 1 \neq 0$, we have

$$\delta a + \delta b \ln W_{\text{Hg}} = 0$$

or

$$\delta a = -\delta b \ln W_{\text{Hg}} \tag{10}$$

When substituting into Eq. 10, we find

$$\delta W(T) = \delta b[W(T) - 1] \ln[W(T)/W_{\text{Hg}}] \tag{11}$$

At the triple point of argon, Eq. 11 satisfies

$$\delta W_{\mathrm{Ar}} = \delta b[W_{\mathrm{Ar}} - 1]\ln[W_{\mathrm{Ar}}/W_{\mathrm{Hg}}]$$

leading to

$$\delta b = \frac{\delta W_{\mathrm{Ar}}}{[W_{\mathrm{Ar}} - 1]\ln[W_{\mathrm{Ar}}/W_{\mathrm{Hg}}]} \tag{12}$$

Substituting it into Eq. 11 yields

$$\delta W(T) = \frac{\delta W_{\mathrm{Ar}}[W(T) - 1]\ln[W(T)/W_{\mathrm{Hg}}]}{[W_{\mathrm{Ar}} - 1]\ln[W_{\mathrm{Ar}}/W_{\mathrm{Hg}}]} \tag{13}$$

which is converted into temperature values as

$$\delta t = \frac{\delta W(T)}{\mathrm{d}W(T)/\mathrm{d}t} = \frac{\delta W_{\mathrm{Ar}}[W(T) - 1]\ln[W(T)/W_{\mathrm{Hg}}]}{[W_{\mathrm{Ar}} - 1]\ln[W_{\mathrm{Ar}}/W_{\mathrm{Hg}}][\mathrm{d}W(T)/\mathrm{d}t]} \tag{14}$$

where δT is an error at any temperature in the subrange. Its approximation is

$$\begin{aligned}\delta T &= \frac{\delta W_{\mathrm{Ar}}[W_{\mathrm{r}}(T) - 1]\ln[W_{\mathrm{r}}(T)/W_{\mathrm{r}}(\mathrm{Hg})]}{[W_{\mathrm{r}}(\mathrm{Ar}) - 1]\ln[W_{\mathrm{r}}(\mathrm{Ar})/W_{\mathrm{r}}(\mathrm{Hg})][\mathrm{d}W_{\mathrm{r}}(T)/\mathrm{d}T]} \\ &= \delta T_{\mathrm{Ar}} \times \frac{[W_{\mathrm{r}}(T) - 1]}{[W_{\mathrm{r}}(\mathrm{Ar}) - 1]} \times \frac{\ln[W_{\mathrm{r}}(T)/W_{\mathrm{r}}(\mathrm{Hg})]}{\ln[W_{\mathrm{r}}(\mathrm{Ar})/W_{\mathrm{r}}(\mathrm{Hg})]} \times \frac{[\mathrm{d}W_{\mathrm{r}}(T)/\mathrm{d}T]_{\mathrm{Ar}}}{\mathrm{d}W_{\mathrm{r}}(T)/\mathrm{d}T}\end{aligned} \tag{15}$$

where $\delta T_{\mathrm{Ar}} = \frac{\delta W_{\mathrm{Ar}}}{[\mathrm{d}W_{\mathrm{r}}(T)/\mathrm{d}T]_{\mathrm{Ar}}}$ represents the error at the triple point of argon in terms of temperature, and $[\mathrm{d}W_{\mathrm{r}}(T)/\mathrm{d}T]_{\mathrm{Ar}}$ represents the derivative of the reference function at the fixed point. The curve of δT is shown as Fig. 1. The maximum error is at the triple point of argon.

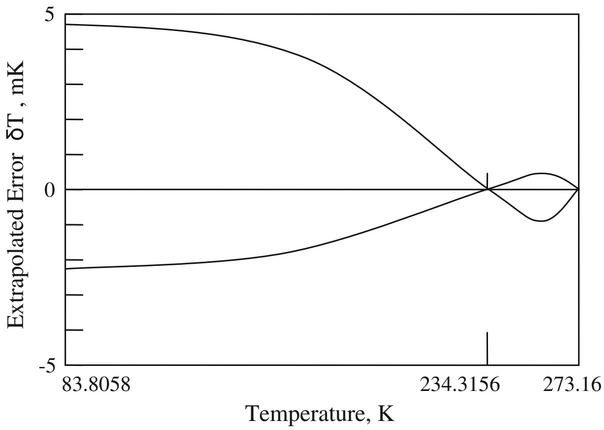

Fig. 1 Curves of δT

6 Conclusion

This article proposes a correlation relation between the resistance ratios of the triple points of argon and mercury and a deviation function which is defined in the range from 83.8058 K to 273.16 K. The function is only determined from calibration values at the triple points of water and mercury. It is a close approximation to the ITS-90 deviation function in the subrange, and the error is less than 5 mK. Using it, we can save the calibration at the triple point of argon.

The correlative relation is similar to that of the resistance ratio at the fixed points of tin, zinc, and aluminum mentioned in Ref. [2]. This shows that the correlation of the fixed points is not accidental and may be a general behavior of SPRTs which needs further research.

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